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ACCURACY EVALUATION OF GAMMA-METHOD FOR DEFLECTION PREDICTION OF PARTIAL COMPOSITE BEAMS

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ABSTRACT: In this paper a precise model is established for deflection prediction of mechanically jointed beams with partial composite action. High accuracy of the proposed method is demonstrated through comparison with a comprehensive finite element (FE) modelling for a timber-concrete partial composite beam. Next, the obtained numerical results are compared with gamma-method, a well-known simplified solution for timber engineers according to the Eurocode 5. Validity and accuracy level of the gamma-method are investigated for various boundary conditions as well as different values of beam length-to-depth ratio, and discussed in details.

KEYWORDS: Partial composite, Shear deformations, Gamma-method, Exact solution.

1 INTRODUCTION

Composite structures made of different construction materials like timber, steel and concrete are increasingly used in construction of building structures. This is due to their inherent benefits that enable providing a better solution in terms of cost, load capacity, environmental impacts, etc.

The cross section of a composite structural element is built from two or more parts that mostly linked by mechanical fasteners; especially for beams made of timber. Perfect bonding between different parts of a composite beam is difficult to be achieved by means of mechanical connectors. Therefore, effect of relative shear slip at the interfaces must be considered, corresponding to partial type of composite interaction.

Effectiveness level of a partial composite beam is often evaluated by comparing the difference between its bending stiffness and that of a similar beam without any connectors (i.e. no composite action), with the maximal possible difference value of bending stiffness (i.e. difference between beams with full- and non-composite actions). A similar efficiency parameter can be defined in terms of the relative deflection.

In Eurocode 5 – annex B, a simplified method has been presented for calculation of the bending stiffness and the loads in a mechanically jointed beam, called gamma-method. It was first developed in 1955 by Möhler [1] for up to 4-layer poles. This law has been integrated in the German standard DIN 102 since 1970. The method is easy and straightforward to be utilised, and has been used in the timber construction industry for a long time. This method is based on the static solution for a simply-supported partial composite beam subjected to distributed sinusoidal transverse load. Therefore, the gamma is not general in nature and cannot be applied to arbitrary boundary and loading conditions.

There have been several studies in literature about static analysis of partial composite beams. Girhammar [2] proposed an analytical method for deflection analysis of partial composite beams based on the Euler–Bernoulli hypothesis, and showed that the error in EC5 for clamped beams can be around 30%. Demarzo and Tacitano [3] studied the static analysis of partial composite beams with T-shape made of a short concrete slab mechanically jointed with a timber rib underneath. They found small difference for deformation and stresses in the simply supported beams subjected to uniformly distributed load when compared with the gamma-method. However, Akasha et al. [4] concluded, in a study on wider timber-concrete composite slabs, that the experimental bending stiffness was at least two times larger than the theoretical bending stiffness according to the gamma-method.

In recent years, some investigators have studied the structural behaviour of partial composite beams including the effect of shear deformations. However, they did not compare the obtained results with those of gamma–method. Nguyen et al. [5] presented an analytical solution for two-layer partial composite beams. Keo et al. [6] presented a numerical solution based on the finite element method (FEM) for multi-layer partial composite beams. Turmo et al. [7] developed an FE model for the analysis of composite beams with partial interaction. Recently, Atashipour et al. [8] developed an exact analytical solution for the buckling of partial composite beams based on different shear deformable models. The concept of partial composite interaction was utilised for deflection analysis of deep composite beams having weak shear webs in [9].

The aim of the present study is to establish a precise analytical model for deflection prediction of partial composite beams including shear deformations and investigate the accuracy level of the gamma-method for various types of edge conditions and different values of the beam length-to-depth ratio. The accuracy and

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validity of the conclusions are verified by comparing the obtained results with those of finite element modelling.

2 MATHEMATICAL MODELLING OF THE PROBLEM

A mathematical model is established here to analyse the two-layer partial composite beams subjected to a uniformly-distributed transverse load with arbitrary boundary conditions.

2.1 BASIC ASSUMPTIONS OF THE MODEL

The present model accounts for shear deformations based on the Timoshenko hypothesis. This would be of importance, especially when a part of beams made of timber which is weak in shear. Only two-layer partial beams are treated in this paper (see Figure 1). Linear elastic state is assumed corresponding to small deformations in the beam parts. Also, a linear slip behaviour is considered at the interface. The common shear correction factor equal to 5/6 is considered, for both of the beam parts.

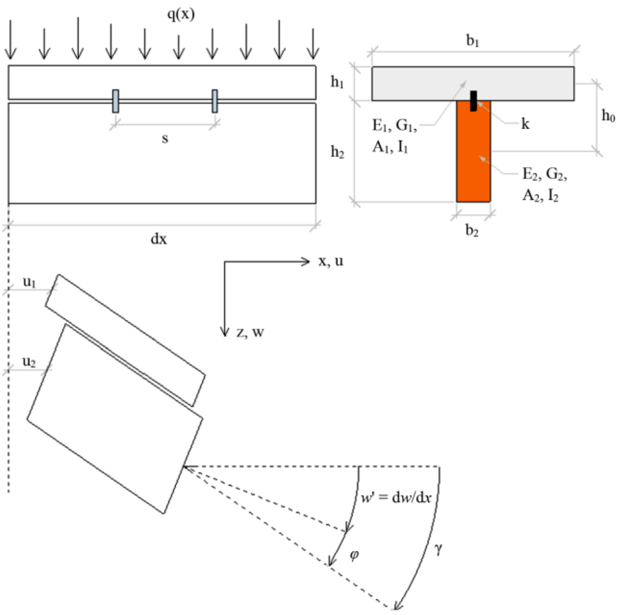

Figure 1: *Kinematic assumptions in the two-layer shear deformable partial composite beam.*

2.2 GOVERNING EQUATIONS OF THE MODEL

To derive the governing equations of the model, the principle of minimum total potential energy is used as

$$\delta(U+V)=0 \tag{1}$$

where the strain energy U and the work done by the external loads V are defined as

$$U=\frac{1}{2}\int_0^L\left\{EA_1u_1'^2+EA_2u_2'^2+EI_1\varphi'^2+EI_2\varphi'^2+K_sGA_1\left(\varphi+w'\right)^2\right.$$
$$\left.+K_sGA_2\left(\varphi+w'\right)^2+k\left(u_2-u_1-h_0\varphi\right)^2\right\}\mathrm{d}x \tag{2.1}$$

$$V=-\int_0^L q(x)w\,\mathrm{d}x \tag{2.2}$$

in which EA_i is the axial stiffness, EI_i the bending stiffness, GA_i the shear stiffness, and u_i the axial displacement in *i*-th member ($i=1,2$). Moreover, w is the deflection, φ is the rotation function, k is the slip modulus, and h_0 is the distance between the centroids of the beam members.

The governing differential equations are obtained as

$$\begin{aligned}
&\delta u_1: EA_1u_1''+k\left(u_2-u_1-h_0\varphi\right)=0\\
&\delta u_2: EA_2u_2''-k\left(u_2-u_1-h_0\varphi\right)=0\\
&\delta\varphi: EI_0\varphi''+h_0k\left(u_2-u_1-h_0\varphi\right)-K_sGA_0(\varphi+w')=0\\
&\delta w: K_sGA_0(w''+\varphi')=-q(x)
\end{aligned} \tag{3}$$

where $EI_0=E_1I_1+E_2I_2$ and $GA_0=G_1A_1+G_2A_2$ are the total non-composite bending stiffness and total shear stiffness, respectively.

2.3 EXACT SOLUTIONS

The exact closed-form solution of the governing Eqs. (3) for the case that a uniformly distributed load of intensity q_0 is applied, are achieved. Details of the solution procedure are given in Appendix A. The obtained closed-form solution for the deflection, axial displacement in different members of the beam as well as the rotation function can be represented as follows

– *Beam deflection:*

$$\begin{aligned}
&w=C_1+C_2x+C_3x^2+C_4x^3+\frac{q_0}{24EI_\infty}x^4\\
&+\left[C_5\cosh(\mathrm{A}_{\mathrm{II}}x)+C_6\sinh(\mathrm{A}_{\mathrm{II}}x)\right]
\end{aligned} \tag{4.1}$$

– *Rotation function:*

$$\begin{aligned}
&\varphi=-C_2-2Cx-3\left(x^2+\frac{2EI_\infty}{K_sGA_0}\right)C_4\\
&-\mathrm{A}_{\mathrm{II}}\left[C_5\sinh(\mathrm{A}_{\mathrm{II}}x)+C_6\cosh(\mathrm{A}_{\mathrm{II}}x)\right]\\
&-\left(\frac{1}{6EI_\infty}x^2+\frac{1}{K_sGA_0}\right)xq_0
\end{aligned} \tag{4.2}$$

– *Axial displacement in upper member:*

$$\begin{aligned}
&u_1=-\frac{EI_0}{h_0EA_1}\left\{C_2+2C_3x+3\left[\left(1-\frac{EI_\infty}{h_0EA_1}\right)x^2+\frac{2EI_\infty}{K_sGA_0}\right]C_4\right\}\\
&-\frac{EI_0\mathrm{A}_{\mathrm{II}}}{h_0EA_1}\left[C_5\sinh(\mathrm{A}_{\mathrm{II}}x)+C_6\cosh(\mathrm{A}_{\mathrm{II}}x)\right]+C_7x+C_8\\
&+\frac{1}{6h_0EA_1}\left[\left(1-\frac{EI_0}{EI_\infty}\right)x^2-\frac{6EI_0}{K_sGA_0}\right]xq_0
\end{aligned} \tag{4.3}$$

– *Axial displacement in lower member:*

$$\begin{aligned}
&u_2=-\left(h_0+\frac{EI_0}{h_0EA_1}\right)\left[C_2+2C_3x+3\left(x^2+\frac{2EI_\infty}{K_sGA_0}\right)C_4\right]\\
&+\frac{3EI_\infty}{h_0EA_1}\left[x^2-\frac{2EA_1}{k}\left(1-\frac{EI_0}{EI_\infty}\right)\right]C_4\\
&+\frac{EI_0\mathrm{A}_{\mathrm{II}}}{h_0EA_2}\left[C_5\sinh(\mathrm{A}_{\mathrm{II}}x)+C_6\cosh(\mathrm{A}_{\mathrm{II}}x)\right]+C_7x+C_8\\
&-\left[\left(h_0+\frac{EI_0}{h_0EA_1}\right)\left(\frac{x^2}{6EI_\infty}+\frac{1}{K_sGA_0}\right)-\frac{x^2}{6h_0EA_1}+\frac{h_0}{EI_0\mathrm{A}_{\mathrm{II}}^2}\right]xq_0
\end{aligned} \tag{4.4}$$

where the coefficient A_{II} is the slip index for the two-layer partial composite beam and is defined as

$$A_{II} = \sqrt{\frac{k}{EI_0}\left(\frac{EA_0 EI_0}{EA_1 EA_2} + h_0^2\right)} = \sqrt{\frac{h_0^2 k}{EI_0\left(1 - EI_0/EI_\infty\right)}} \tag{5}$$

In Eq. (5), EI_∞ is the bending stiffness corresponding to the full-composite action case, and is defined in terms of the bending stiffness of a non-composite case, EI_0, as follows

$$EI_\infty = \left(1 + \frac{EA_1 EA_2}{EA_0 EI_0} h_0^2\right) EI_0 \tag{6}$$

Obviously, $EA_0 = E_1 A_1 + E_2 A_2$ is the total axial stiffness of the beam. In Eqs. (4.1) through (4.4), the eight unknown constant coefficients $(C_i,\ i = 1, 2, ..., 8)$ are to be determined from satisfaction of arbitrary boundary conditions at the partial beam ends. We study different combinations of classical end conditions. For instance, pinned-pinned, Clamped-Free, Clamped-Clamped, etc. The boundary equations for each of them in the partial composite shear deformable beam are represented in the form:

Pinned – Pinned:

$$\text{at } x = 0, L:\ w = \varphi' = u_1' = u_2' = 0 \tag{7.1}$$

Clamped – Free:

$$\begin{aligned} &\text{at } x = 0:\ w = \varphi = u_1 = u_2 = 0 \\ &\text{at } x = L:\ w' + \varphi = \varphi' = u_1' = u_2' = 0 \end{aligned} \tag{7.2}$$

Clamped – Pinned:

$$\begin{aligned} &\text{at } x = 0:\ w = \varphi = u_1 = u_2 = 0 \\ &\text{at } x = L:\ w = \varphi' = u_1' = u_2' = 0 \end{aligned} \tag{7.3}$$

Clamped – Clamped:

$$\text{at } x = 0, L:\ w = \varphi = u_1 = u_2 = 0 \tag{7.4}$$

Each of the above end conditions leads to a set of eight algebraic equations for determining the aforementioned eight unknown constants.

2.4 FORMULAE FOR MAXIMUM DEFLECTION

As mentioned in the previous section, four different combinations of classical end conditions, namely pinned-pinned, clamped-free, clamped-clamped and clamped-pinned, are considered in this study. The exact maximum deflection of the aforementioned partial beams, when they are subjected to the uniformly distributed load of intensity q_0, are obtained in a simple dimensionless form for practical applications. To this end, the following dimensionless parameters are introduced:

$$\begin{aligned} \bar{w} &= \frac{EI_\infty}{q_0 L^4} w \\ \eta &= \frac{EI_\infty}{L^2 K_s GA_0} \\ \omega_{II} &= A_{II} L \end{aligned} \tag{8}$$

Evidently, η represent the contribution of shear deformations in the deflection, and ω_{II} indicates the slip level (see Eq. (5)). Therefore, the exact dimensionless deflection for different cases are expressed as

Pinned – Pinned:

$$\bar{w}_{\max} = \frac{5}{384} + \frac{1}{8}\eta + \frac{\alpha}{\omega_{II}^4}\left(\frac{1}{8}\omega_{II}^2 + \operatorname{sech}(\omega_{II}/2) - 1\right) \tag{9.1}$$

Clamped – Free:

$$\bar{w}_{\max} = \frac{1}{8} + \frac{1}{2}\eta + \frac{\alpha}{\omega_{II}^4}\left(\frac{1}{2}\omega_{II}^2 - \omega_{II}\tanh(\omega_{II}) - \operatorname{sech}(\omega_{II}) + 1\right) \tag{9.2}$$

Clamped – Clamped:

$$\bar{w}_{\max} = \frac{1}{384} + \frac{1}{8}\eta + \frac{\alpha}{2\omega_{II}^3}\left(\frac{1}{4}\omega_{II} - \coth(\omega_{II}/2) + \operatorname{csch}(\omega_{II}/2)\right) \tag{9.3}$$

where the coefficient $\alpha \geq 0$ indicates the level of difference between bending stiffness in a full-composite beam and that in the same beam with non-composite interaction, and is defined as

$$\alpha = \frac{EI_\infty}{EI_0} - 1 \tag{10}$$

It can be deduced from Eqs. (9) that the maximum deflection in a partial composite beam is comprised of three distinguished terms namely, deflection due to the normal bending deformation, shear deformation and that due to the slip at the interlayers. In Eqs. (9.1-3), the first constant term is the dimensionless deflection based on the Euler-Bernoulli theory in a full-composite beam.

3 FINITE ELEMENT MODELLING

In order to validate the established model and solutions, the deflection of the two-layer composite beams with partial shear connection is calculated using the finite element software RFEM from Dlubal. Both layers have been discretized with 100 mm long beam elements. The partial shear connection between the two layers has been modelled with discrete springs connected rigidly to the beam elements every 100 mm, as shown in Figure 2.

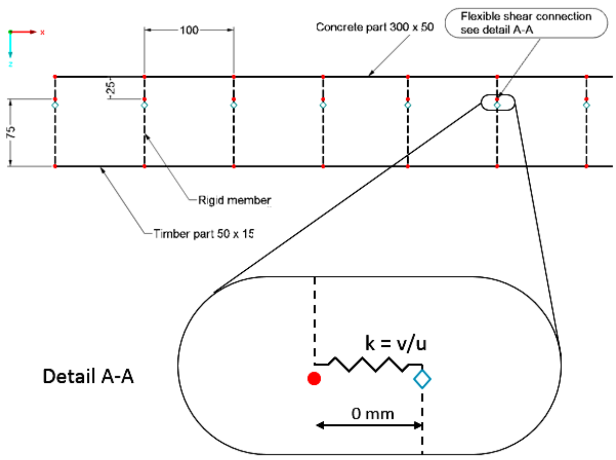

Figure 2: *Finite element model for the partial composite beam.*

The geometrical configuration is illustrated in a sketch in Figure 3; and the mechanical properties and dimensions are listed in table 1.

Table 1: *Geometrical and mechanical characteristics for the considered concrete-timber partial composite beam.*

Part 1: Concrete		
Height h_1	50	mm
Width b_1	300	mm
Modulus of elasticity E_1	12 000	MPa
Shear modulus G_1	5 000	MPa
Shear connectors		
Shear connector stiffness k	50.1	N/mm/mm
Part 2: Structural Timber C16		
Height h_2	150	mm
Width b_2	50	mm
Modulus of elasticity E_2	8 000	MPa
Shear modulus G_2	500	MPa

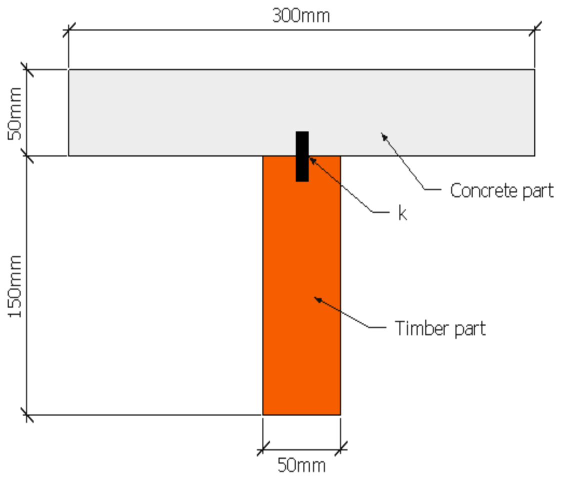

Figure 3: *Cross section of the considered two-layer concrete-timber partial composite beam.*

4 NUMERICAL RESULTS AND DISCUSSION

Using the developed analytical solution as well as the FE modelling, a comparative study is conducted in this section between the obtained results and those of the gamma-method for the partial composite beams subjected to uniformly-distributed transverse loading. For the sake of conducting comparative numerical results, a timber-concrete partial composite beam is considered (details of geometrical dimensions and material properties are given in Table 1 and Figure 3). The results are also compared with those reported [2] on the basis of Euler partial composite hypothesis. The comparative numerical results are presented in Table 2 for three different values of length to thickness ratio. Also, four different combination of classical edge conditioned are considered, namely: free-clamed, pinned-pinned, pinned-clamped and clamped-clamed.

It can be seen from the results of this table that the present method gives closest results to those of FE modelling when compared to other methods. Also, the error of the results based on Euler partial composite hypothesis grows when the beam length to thickness ratio decreases. However, the predicted deflection of the partial composite beam is, for all cases, in very good agreement with that based on the FE modelling.

Table 2: *Maximum deflection (mm) (Uniformly distributed load q_0=1 kN/m).*

B.Cs	Source	L/h_{Total}		
		20	10	5
F–C	Girhammar [2]	–	4.76	–
	Present	60.56	4.791	0.4526
	FE modelling	60.67	4.801	0.4513
	Gamma-method	58.69	4.550	0.431
	(%Diff-1)	-3.3%	-5.2%	-4.5%
	(%Diff-2)	-3.1%	-5.0%	-4.8%
P–P	Girhammar [2]	7.56	–	–
	Present	7.587	0.7244	0.0684
	FE modelling	7.546	0.711	0.068
	Gamma-method	7.584	0.719	0.067
	(%Diff-1)	+0.5%	+1.1%	-1.5%
	(%Diff-2)	+0.0%	-0.7%	-2.0%
P–C	Girhammar [2]	4.01	–	–
	Present	4.039	0.3924	0.0335
	FE modelling	4.054	0.395	0.0345
	Gamma-method	3.538	0.345	0.0300
	(%Diff-1)	-13%	-13%	-13%
	(%Diff-2)	-12%	-12%	-11%
C–C	Girhammar [2]	2.32	–	–
	Present	2.354	0.2214	0.0180
	FE modelling	2.384	0.228	0.0185
	Gamma-method	1.934	0.188	0.015
	(%Diff-1)	-19%	-18%	-19%
	(%Diff-2)	-18%	-15%	-17%

In order to evaluate the accuracy level of the gamma-method, two percentage difference parameters are defined as follows:

$$\%\text{Diff-1} = \frac{w_{\max}^{\text{Gamma}} - w_{\max}^{\text{FE}}}{w_{\max}^{\text{FE}}} \times 100\% \qquad (11.1)$$

$$\%\text{Diff-2} = \frac{w_{\max}^{\text{Gamma}} - w_{\max}^{\text{Exact}}}{w_{\max}^{\text{Exact}}} \times 100\% \qquad (11.2)$$

Corresponding percentage difference values are provided in Table 2. It can be concluded from the results of Table 2 that, except for the pinned-pinned case, gamma-method significantly underestimates the maximum deflection of the partial beams. The largest error of the gamma-method is related to the clamped-clamped case, which is around 20%. It is worth mentioning that, the error of the gamma-method would be even more when all of the beam members are made of wood/composites which have a low shear rigidity. Therefore, it seems necessary that the current version of the gamma-method in EC5 to be revised for partial composite beams with non-simply supported ends.

5 CONCLUSIONS

Exact deflection analysis of partial composite beam was treated in this paper. The effects of shear deformations were incorporated into the analysis. The accuracy of the gamma-method and the influence of neglecting shear deformations (i.e. Euler hypothesis) were studied for different types of beam end conditions. It was concluded that necessity of using a modification coefficient is essential for the gamma-method for deflection prediction of partial beams having the end conditions other than the simply supported.

APPENDIX A: DETAILS OF SOLUTION PROCEDURE

Eq. (3.1) can be rewritten as:

$$u_2 = -\frac{E_1A_1}{k}u_1'' + u_1 + h_0\varphi \qquad \text{(A.1)}$$

Substituting Eq. (A.1) into Eq. (3.2) leads to the following equation

$$\frac{E_1A_1}{k}u_1^{(4)} - \frac{EA_0}{E_2A_2}u_1'' - h_0\varphi'' = 0 \qquad \text{(A.2)}$$

where

$$EA_0 = E_1A_1 + E_2A_2 \qquad \text{(A.3)}$$

Differentiating Eq. (3.3) one time and eliminating the term $\kappa_s GA_0(\varphi' + w'')$ form Eq. (3.4) yield

$$EI_0\varphi''' + h_0k\left(u_2' - u_1' - h_0\varphi'\right) = -q(x) \qquad \text{(A.4)}$$

Differentiating Eq. (A.1) one time and substituting into Eq. (A.4) results in

$$u_1''' = \frac{1}{h_0E_1A_1}\left[EI_0\varphi''' + q(x)\right] \qquad \text{(A.5)}$$

Differentiating Eq. (A.2) one time, and substituting Eq. (A.5) and its derivatives, gives

$$\varphi^{(5)} - \left[\frac{EA_0}{E_2A_2}\frac{k}{E_1A_1} + \frac{h_0^2k}{EI_0}\right]\varphi''' + \frac{1}{EI_0}q''(x) - \frac{1}{EI_0}\frac{EA_0}{E_2A_2}\frac{k}{E_1A_1}q(x) = 0 \qquad \text{(A.6)}$$

Also, Eq. (3.4) can be rewritten as

$$\varphi' = -w'' - \frac{1}{K_sGA_0}q(x) \qquad \text{(A.7)}$$

Substitution of Eq. (A.7) into Eq. (A.6), an uncoupled equation in terms of deflection function is obtained as follows

$$w^{(6)} - \mathrm{A}_{\mathrm{II}}^2 w^{(4)} = -\frac{1}{\kappa_s GA_0}q^{(4)} + \left(\frac{1}{EI_0} + \frac{\mathrm{A}_{\mathrm{II}}^2}{K_sGA_0}\right)q'' - \frac{\mathrm{A}_{\mathrm{II}}^{2}}{EI_\infty}q \qquad \text{(A.8)}$$

where

$$\mathrm{A}_{\mathrm{II}} = \sqrt{\frac{1}{EI_0}\left(\frac{EA_0EI_0}{E_1A_1E_2A_2} + h_0^{2}\right)k} \qquad \text{(A.9)}$$

Using the definition of the bending stiffness corresponding to the full composite action in the form:

$$EI_\infty = \left(1 + \frac{E_1A_1E_2A_2}{EA_0EI_0}h_0^{2}\right)EI_0 \qquad \text{(A.10)}$$

one can rewrite Eq. (A.9) in a more efficient form as

$$\mathrm{A}_{\mathrm{II}} = \sqrt{\frac{h_0^2k}{EI_0\left(1 - EI_0/EI_\infty\right)}} \qquad \text{(A.11)}$$

For a uniformly distributed load of intensity q_0, Eq. (A.8) is simplified as

$$w^{(6)} - \mathrm{A}_{\mathrm{II}}^2 w^{(4)} = -\frac{\mathrm{A}_{\mathrm{II}}^2}{EI_\infty}q_0 \qquad \text{(A.12)}$$

The complete solution of Eq. (A.12) is given as

$$w(x) = c_1 + c_2x + c_3x^2 + c_4x^3 + c_5\cosh(\mathrm{A}_{\mathrm{II}}x) + c_6\sinh(\mathrm{A}_{\mathrm{II}}x) + \frac{q_0}{24EI_\infty}x^4 \qquad \text{(A.13)}$$

From integrating Eq. (A.7) we will obtain

$$\varphi = -w' - \frac{q_0}{K_sGA_0}x + c_7 \qquad \text{(A.14)}$$

and consequently,

$$\varphi(x) = -c_2 - 2c_3x - 3c_4x^2 - \frac{q_0}{6EI_\infty}x^3 - \frac{q_0}{K_sGA_0}x + c_7 - \mathrm{A}_{\mathrm{II}}\left[c_5\sinh(\mathrm{A}_{\mathrm{II}}x) + c_6\cosh(\mathrm{A}_{\mathrm{II}}x)\right] \qquad \text{(A.15)}$$

Integrating Eq. (A.5) for the uniform load of intensity q_0 three times yields

$$u_1 = \frac{1}{h_0EA_1}\left[EI_0\varphi + \frac{1}{6}q_0x^3\right] + c_8x^2 + c_9x + c_{10} \qquad \text{(A.16)}$$

Now, we substitute Eq. (A.15) into Eq. (A.16) and obtain

$$u_1 = c_8x^2 + c_9x + c_{10} + \frac{EI_0}{h_0E_1A_1}\left\{ -c_2 - 2c_3x - 3c_4x^2 - \mathrm{A}_{\mathrm{II}}\left[c_5\sinh(\mathrm{A}_{\mathrm{II}}x) + c_6\cosh(\mathrm{A}_{\mathrm{II}}x)\right] - \frac{q_0}{6EI_\infty}x^3 + \left(\frac{1}{6EI_0}x^3 - \frac{1}{\kappa_sGA_0}x\right)q_0\right\} \qquad \text{(A.17)}$$

From the above ten constant coefficients, only eight of them should be independent and enough for satisfaction of the boundary conditions. By substitution of the obtained solutions into the governing equations, two dependency equations between the constants are obtained as follows

$$c_7 = -\frac{6EI_\infty}{K_sGA_0}c_4, \qquad c_8 = \frac{3EI_\infty}{h_0E_1A_1}c_4 \qquad \text{(A.18)}$$

The remaining eight constants will be determined from the arbitrary boundary conditions.

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